Some Practical Problems Related With Bus Transport System By Optimization Methods

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Abstract -In the paper, a school bus routing problem, its mathematical models and solution methods are investigated. The aim of the study is to search for school bus routing problem and its solution method and to apply them for a sample case study. The case study concerns the routing and scheduling of school buses in an exemplary, well-recognized school located in one of Patna. The problem is to find a series of school bus routes that ensure the service is provided equitably to all eligible students. Because of the NP-hardness of the school bus routing problem, it is solved using some heuristic optimization method using real data from the considered exemplary school. The aim is to increase bus utilization and to reduce transportation times for students, while maintaining on-time delivery of students to the school. Although the problem under consideration is one of the earliest logistics problems solved using methods of operations research, remains valid and is the subject of research, as evidenced by numerous contemporary publications, presenting new methods for the formal specification and solution of the problem.

Keywords: *School bus routing problem, scheduling, heuristic, optimization*

INTRODUCTION-Vehicle Routing Problem (VRP) is a typical logistic problem which has been widely studied on by many resarchers. It is also a classic example of the logistic problem solution of which is successfully supported by operational research methods, particularly mathematical modeling and optimization. VRP can be described as a problem of determining efficient routes for a fleet of vehicles in order to deliver or collect products from depots to a set of customers. Vehicle Routing Problem turns to be a relatively difficult problem when the number of inputs increases. This type of problem has been widely used in daily life so that trans-portation and distribution costs reflect as important cost elements into companies, enterprises, firms, schools etc. Human beings meet with VRP in different areas in daily life, for example various types of food and drink, clothing, heating material transportation, and garbage collection, postal service, personnel and student trans-portation. In this point of view, Vehicle Routing Problem separates into two main areas: human and freight transportation.

School bus routing is a version of the traveling salesman problem, commonly referred to the category of vehicle routing problems, either with or without time window constraints. In addition to the numerous studies that addressed the vehicle routing problem, various software methods have been developed that can utilized to minimize the operating cost. Three factors make school bus routing unique (Spasovic et al., 2001): efficiency (the total cost to run a school bus), effectiveness (how well the demand for service is satisfied) and equity (fairness of the school bus for each student).

The purpose of the considerations set out in the paper is to present a verbal and formal description of the School Bus Routing Problem (SBRP), to put forward some new suggestions for practical and effective solution and to apply them for a case study based on well-recognized school located in one of Patna.

There are three objectives (measures) most commonly used for the SBRP (Spasovic at el., 2001): efficiency (the total cost to run a school bus), effectiveness (how well the demand for service is satisfied) and equity (fairness of the school bus for each student).

MATHEMATICAL FORMULATION OF THE SBRP-The mathematical models for SBRP are developed for various configurations. SBRP is usually formulated as mixed integer programming (MIP) or nonlinear mixed integer programming (NLMIP) models. However, most of them have not been used directly to solve the problems, and they are often not for the whole problem but for the SBRP parts (Park & Kim, 2010). In accordance with previous remarks the school bus routing has two main routing issues – assigning students to bus stops and routing the buses to the bus stops. In this section the SBRP consists of finding an optimizing collection of some simple bus routes corresponding to buses selected from an available bus fleet will be considered. The school bus routing problem can be presented conceptually as a linear cost minimization problem, in which the objective function is the total operating

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cost (Spasovic at el., 2001). The objective function can be minimized subject to a series constraints listed below:

1. each selected bus performs exactly one route,
2. each bus stop is allocated to only one bus,
3. every route must have at least one stop,
4. each route begins at school and arrives at school,
5. each selected bus stop can be visited by only one selected bus,
6. the number of students on each selected bus must not exceed the bus capacity,
7. accessible bus fleet has known number of the same buses,
8. the number of buses leaving the school must equal the number of buses re-turning to the school,
9. the travel time of each selected bus must not exceed the time duration allowed.

Let $\boldsymbol{I}$ mean a set of allowed bus route numbers, $\boldsymbol{I}=\{1,2,3,...,i,...,I\}$. Let $\boldsymbol{J}$ mean a set of allowed bus stop numbers, $\boldsymbol{J}=\{1,2,3,...,j,...,J\}$, where the bus number *1*

corresponds to the school. The set of bus stops, including the school, creates some bus stop network. Formally, a bus stop network can be defined as an undirected graph $G=(\boldsymbol{J}, \boldsymbol{E})$ with distances on the edges $d: E$ ® $\boldsymbol{R}^+$, where $\boldsymbol{R}^+$ is set of positive

real numbers, and set $\boldsymbol{J}$ representing the bus stops is defined as follows: $(i, j) \in \boldsymbol{E}$,

if there is edge in G, connecting the vertices i, j, i.e. if after starting from the *i-th* stop as a next bus stop can be the *j-th* stop. It is convenient to represent the bus stop network G by means of a square matrix $S = [\, s_{ij} \,]_{J \times J}$, where

$$s_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \boldsymbol{E}, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Additionally, we are given a bus capacity C Î $\boldsymbol{N}$, where $\boldsymbol{N}$ is the set of natural numbers. In turn, in accordance with the constraints *1)*, *2)* and *5)* a bus number $i \in \boldsymbol{I}$ uniquely identifies the bus route. According to constraint *7)* each bus $i \in \boldsymbol{I}$ has the same capacity C. Let $\boldsymbol{L}$ mean a set of bus route numbers. In accordance with previous remarks we have $\boldsymbol{L} \subseteq \boldsymbol{E}$, wherein $\boldsymbol{L} = \boldsymbol{E}$ if all buses were used to transport students. We assume that it is also known a square distance matrix

$D = [\, d_{ij} \,]_{J \times J}$, where element $d_{ij} \in \boldsymbol{R}+$ is equal to the distance between *i-th* and *j-th* bus stops, if $(i, j) \in \boldsymbol{E}$. Let $X = [\, x_{ijk} \,]_{J \times J \times I}$ mean a three-dimensional matrix, where element x_{ijk} is defined as follows

$$x_{ijk} = \begin{cases} 1 & \text{if } s_{ij} = 1 \text{ and bus stops } i \text{ and } j \text{ belongs to route of bus } k, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

The matrix X will be called in further consideration as a school transport strategy. As a consequence of assumptions that have been determined and the definition (2) we have

$$\sum_{i \in J} x_{ijk} = \begin{cases} 1 & \text{if } i\text{-th bus stop is catered by bus route } k, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

For the purpose of formulating of the exemplary SBRP we assume the following additional descriptions:

T_{max} - maximum time available for the bus to pick up students on a route, n_i - number of students to be picked up at *i-th* bus stop,
K - operating cost for a bus (e.g. in € per bus-hour), V - average speed for a bus (e.g. in kilometers per hour).

The objective is to minimize total operating cost $K(X)$ for a school transport strategy X that can be formulated as

$$K(X) = K \sum_{i \in I} T_i(X), \quad (4)$$

where

$T_i(X)$ is a time taken for bus k to pick up students on *k-th* bus route and drop students at school (node *1*) can be roughly computed (excluding time to pick up students on *k-th* bus route and drop students at the school) as

$$T_i(X) = \sum_{i \in J} \sum_{j \in J} (x_{ijk} \cdot d_{ij} / V). \quad (5)$$

On the base earlier assumptions and descriptions that were taken we can formulate the following formal optimization problem of the SBRP class:

$$\text{minimize } K(X) = K \sum_{i \in I} T_i(X), \quad (6)$$

subject to the following set of constraints:

$$T_i(X) \leq T_{max},\ i \in \boldsymbol{I}, \tag{7}$$

$$\sum\sum (x_{ijk}\ n_i) \leq C,\ k \in \boldsymbol{I}, \tag{8}$$

$i \in J\ j \in J$

$$\sum_{k \in I}\sum_{j \in J} x_{ijk} = 1\ \ i \in \boldsymbol{J} - \{1\},$$

(9)

$$\sum_{k \in I}\sum_{i \in J} x_{ijk} = 1\ \ j \in \boldsymbol{J} - \{1\},$$

(10)

$$\sum_{k \in I}\sum_{j \in J} x_{1jk} \geq 1,$$

(11)

$$\sum_{k \in I}\sum_{i \in J} x_{i1k} \geq 1,$$

(12)

$$\sum_{j \in J} x_{1jk} \leq 1,\ k \in \boldsymbol{I}, \tag{13}$$

$$\sum_{j \in J} x_{j1k} \leq 1,\ k \in \boldsymbol{I}, \tag{14}$$

$$\sum_{j \in J} x_{ijk} = \sum_{j \in J} x_{jik},\ k \in \boldsymbol{I},\ i \in \boldsymbol{I}. \tag{15}$$

Constraints (7)-(16) have following interpretation:

• constraint (7) is to guaranty that the every bus do not travel over the time allowed,

• constraint (8) is to guaranty that every bus can take allowed number of students (capacity constraint),

• constraint (9) – (10) guaranty that every bus stop belongs to one bus route,

• constraint (11) – (12) guaranty that there are at least one bus route,

• constraint (13) – (14) guaranty that every route is realized by exactly one bus,

• constraint (15) is to guaranty the equality in the number of buses leaving from and arriving to school.

NUMERICAL EXAMPLE-In this section an example of formulation and solving the School Bus Route Prob-lem will be presented. An example is based on the reality of one the Patna junior high school. The school must provide transportation of their students who use the designated bus stops in the area. The structure of bus stop network illustrates graph *G*, as shown in Figure 1.

The school under investigation has $I=3$ buses the same capacity $C=30$. Thus we have $\boldsymbol{I} = \{1,2,3\}$. According to the bus stops structure shown in Figure 1 the set of
bus stop number, the set

$\boldsymbol{E} = \{(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,3),(3,1),(3,2),(3,4),(4,1),(4,5),$ of graph edges have form $\boldsymbol{J} = \{1,2,3,...,j,...6,),(4,3\}$,

$(5,1),(5,4),(5,6),(6,1),(6,5)\}$.

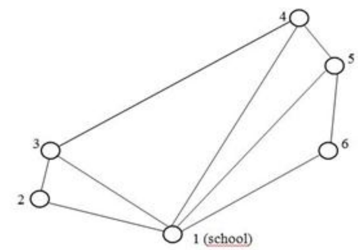

Fig. 1 The structure of bus stop network

The values of other parameters required to formulate and solve SBRP for the analysed case are defined as follows:

• maximum time available for the bus to pick up students on a route, $T_{max} = 0{,}75$ hour,

• number n_i of students to be picked up at *i-th* bus stop: $\boldsymbol{n} = (0,15,9,6,12,8)$,

• operating cost for a bus, $K = 50$ € per bus-hour,

• average speed for a bus, $V=60$ kilometers per hour.

The matrices *S* defined by (1) and *D* representing distances between *i-th* and *j-th* bus stops, $i, j \in \boldsymbol{J} = \{1,2,3,...,j,...6,\}$ have forms

$$S = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 7{,}2 & 9{,}5 & 17 & 15 & 10 \\ 7{,}2 & 0 & 5 & 0 & 0 & \\ 9{,}5 & 5 & 0 & 15 & 0 & \\ 17 & 0 & 11{,}3 & 0 & 4{,}5 & \\ 15 & 0 & 0 & 4{,}5 & 0 & \\ 10 & 0 & 0 & 0 & 9 & \end{bmatrix}$$

Generally speaking, the SBRP, even in its simplest form, is a complex and complicated. It can be proved that the combined problem of bus stop selection and bus route generation for a single school is an NP-hard problem. The bus route generation problem with vehicle capacity and maximum riding time constraints corresponds to a capacitated and distance constrained problem, which is known as an NP-hard problem (Bektas & Elmastas, 2007). Because of its complication only a few papers adopted exact approaches to solve the parts dealing on SBRP.

Due to its difficulty most studies prefer heuristic approaches rather than exact approaches (Park & Kim, 2010). Heuristics are defined as a set of rules that are being followed in solving complex pronlems. A few heuristic methods have been developed to provide the optimization for vehicle routing and scheduling problems that have time window constraints. One of the most known heuristic used to determine the solution of SBRP heuristic proposed by Clark and Wright, known as time saving heuristic (Clark & Wright, 1964).

Due to the small size of the analyzed example SBRP problem, finding its solution is relatively simple. Using any optimization package, e.g. *Mathematica*, gives the following optimal solution of the problem (6) - (15):

- number of bus routes, $J=2$,
- matrix $X^*= [x_{ijk}]_{6\times6\times2}$, that maximizes objective function (4), for each of the two buses, has the form

$$X_1^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad X_2^* = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- optimal bus routes:
  1) 1-2-3-1,
  2) 1-4-5-6-1;
- optimal operating cost of bus routes:

$T_1(X^*) = 0,36$ hour, $T_2(X^*) = 0,68$ hour,

$$K(X^*) = 50 \cdot (0,36 + 0,68) = 52 \text{ €}.$$

It is noteworthy that in the analyzed problem can be identified route 1-2-3-4-5-6-1 requiring of using only one bus, for which total operating cost is 42.5€. However, as can easily noted, this route should not be allowed to set routes, due to the failure constraints (7) and (8).

CONCLUSION-In this paper, the SBRP with regard to various aspects of the problem is described. Although the problems of the SBRP class are one of the earliest logistics problems solved using methods of operations research, they remain valid and are the subject of research, as evidenced by numerous contemporary publications. The paper presents an example of mathematical formulation and solving the problem of optimizing the school transportation system in one of the Patna junior high school. The small size of the problem made it possible to determine the exact solution.

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